| REPORT DOCUMENTATION PAGE | Form Approved OMB NO. 0704-0188 |
|---|---|

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comment regarding this burden estimates or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

| 1. AGENCY USE ONLY (Leave blank) | 2. REPORT DATE March 18, 1998 | 3. REPORT TYPE AND DATES COVERED Final technical report 1/20/93-7/19/97 |
|---|---|---|

| 4. TITLE AND SUBTITLE | 5. FUNDING NUMBERS |
|---|---|
| Modal Analysis Techniques for Nonlinear Large-Scale Structural Systems | DAAH04-93-G-0035 |

**6. AUTHOR(S)**

Christophe Pierre and Steven W. Shaw

| 7. PERFORMING ORGANIZATION NAMES(S) AND ADDRESS(ES) | 8. PERFORMING ORGANIZATION REPORT NUMBER |
|---|---|
| The University of Michigan Division of Research Development and Administration 3003 South State Street Ann Arbor, MI  48109-1274 | |

| 9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES) | 10. SPONSORING / MONITORING AGENCY REPORT NUMBER |
|---|---|
| U.S. Army Research Office P.O. Box 12211 Research Triangle Park, NC 27709-2211 | ARO 30659.5-EG |

**11. SUPPLEMENTARY NOTES**

The views, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy or decision, unless so designated by other documentation.

| 12a. DISTRIBUTION / AVAILABILITY STATEMENT | 12 b. DISTRIBUTION CODE |
|---|---|
| Approved for public release; distribution unlimited. | 19991215 030 |

**13. ABSTRACT (Maximum 200 words)**

The objective of this research was to develop, and to implement computationally, modal analysis methodologies for large-scale, complex, nonlinear structures. These methods are based on nonlinear modes of vibration defined and constructed in terms of invariant manifolds. The motivation for the research stems from the fact that the dynamics of nonlinear structures are typically decomposed in terms of the linearized system's modes, often yielding poor modal convergence and too large reduced-order models.

During this grant, major theoretical advances were made on the fundamentals of modal analysis for nonlinear systems. Novel constructive methodologies were formulated and validated for single- and multi-mode motions of nonlinear systems. Exact optimal reduced-order models were developed for free response nonlinear modal analysis, and approximate ones were proposed for the forced response case. All these methods share a common foundation, namely the groundbreaking definition, by the principal investigators, of nonlinear modes of vibration in terms of invariant manifolds in the system's phase space.

The methods developed were validated and their effectiveness was demonstrated for several nonlinear structural systems such as beams, and results showed nonlinear modal analysis to be significantly more accurate and economical than classical linear modal analysis.

| 14. SUBJECT TERMS | | 15. NUMBER IF PAGES 8 |
|---|---|---|
| Vibration, nonlinear dynamics, structural systems, modes of vibration, modal analysis, reduced-order models, nonlinear modes | | 16. PRICE CODE |

| 17. SECURITY CLASSIFICATION OR REPORT UNCLASSIFIED | 18. SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED | 19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED | 20. LIMITATION OF ABSTRACT UL |
|---|---|---|---|

# FINAL TECHNICAL REPORT

## *for*

## US Army Research Office Grant

## Modal Analysis Techniques for Nonlinear Large-Scale Structural Systems

**Proposal number:** P-30659-EG

**Grant number:** DAAH04-93-G-0035

**Period covered by report:** January 20, 1993—July 19, 1997

**Date submitted:** March 18, 1998

## STATEMENT OF PROBLEM

The basic objective of this research was the development of modal analysis methodologies for non-linear large-scale structural systems, resulting in the systematic generation of optimal reduced-order models for vibration response prediction, attendant reliability assessment, and high fidelity simulation of mechanical structures, such as ground vehicle and rotorcraft structures. The importance of the subject stems from the fact that a traditional linear modal analysis of a nonlinear structure yields undesirably large models, because of the two-way exchange of energy, or contamination, between sets of modes. Contamination is caused by nonremovable coupling among the linear modes through the nonlinearities, resulting in deteriorated modal convergence. In the nonlinear modal analysis developed in this project, contamination between modeled and unmodeled modes was automatically eliminated, thereby allowing one to construct the smallest possible, uncontaminated reduced-order model to characterize the complex dynamic behavior of nonlinear structures to a prescribed accuracy. Removal of modal contamination was achieved by choosing the trial functions to be the *nonlinear normal modes* of the structure, rather than the linear ones. At the heart of this methodology is the novel definition of nonlinear normal modes as invariant motions for the nonlinear system, henceforth inherently embedding the effects of nonlinearities in them and allowing one to eliminate precisely those nonlinear coupling terms that cause unwanted exchange of energy between modeled and unmodeled modes.

# SUMMARY OF MOST IMPORTANT RESULTS

During the past grant years, major theoretical advances were made on the fundamentals of modal analysis for nonlinear systems. Novel constructive methodologies were formulated and validated for single- and multi-mode motions of nonlinear systems. Exact optimal reduced-order models were developed for free response nonlinear modal analysis, and approximate ones were proposed for the forced response case. All these methods share a common foundation, namely the groundbreaking definition, by the principal investigators (PIs), of nonlinear modes of vibration in terms of invariant manifolds in the system's phase space.

Research during the grant years proceeded along four principal lines: (1) the construction of individually invariant nonlinear normal modes for discrete, continuous, damped, and gyroscopic structural systems, (2) the generation of multi-nonlinear mode reduced-order models for general free motions, (3) the development of approximate nonlinear modal analysis and associated reduced-order model reduction procedures for forced response problems and systems with damping, and (4) the application of the definition of normal modes in terms of invariant manifolds to the special case of linear systems. The main results obtained are described below.

## Individually Invariant Normal Modes

Nonlinear normal modes were defined in terms of invariant manifolds, and a constructive technique for obtaining local approximations of them in terms of asymptotic series was developed for both discretized and continuous systems. The nonlinear differential equation governing the dynamics of the system undergoing a normal mode motion was derived, and was shown to be at a higher level of approximation than the invariant manifold itself. It was also found that the equations for the coefficients of the nonlinear modes are linear, making the procedure quite practical for large-scale systems and attractive for implementation in computational structural dynamics codes. The invariance properties of the individual nonlinear modes were verified by numerical simulation on several study cases. An illustration of an individual nonlinear normal mode is given in Fig. 1. The results of this study were presented in a series of papers published in the *Journal of Sound and Vibration, Nonlinear Dynamics*, and other refereed journals [1-9].

## Multi-Nonlinear Mode Reduced-Order Models

Multi-nonlinear mode reduced-order models were constructed for general free motions. These higher-dimensional invariant manifolds were shown to capture all interactions between the modeled modes, while preventing contamination from the unmodeled ones. Multi-mode models represent the minimal set of equations necessary to describe completely the motion of the system at a given order of approximation. Results indicate that nonlinear modal analysis based on invariant manifold models requires many fewer modes than standard linear modal analysis to achieve a desired accuracy, and hence may

yield considerable computational savings. Figure 2 provides an illustration of this finding. Also, it was shown that the accuracy of the dynamics of multi-mode reduced-order models can be increased by increasing the order of the approximation of the invariant manifolds rather than by adding modes to the model, that is, rather than by increasing the number of simulated equations, as is typically done in linear modal analysis. The results obtained from the study of multi-mode manifolds were published in references [10] and [11]. (Further journal publications on this topic are forthcoming, based on the results given in [10].)

### *Systems with Internal Resonances*

The nonlinear modal analysis developed places no restrictions on the possible interactions between the resulting coupled, non-linear modal oscillators and, as such, was shown to be perfectly suited for systems featuring internal resonances. These are systematically accounted for by the procedure, such that when a legitimate set of nonlinear modes has been modeled (*i.e.*, no internally resonant mode has been overlooked up to the order considered), the internal resonances are "transparent" and automatically accounted for without further work. Furthermore, if some modeled nonlinear modes are internally resonant with a non-modeled mode, the procedure automatically detects the anomaly with a singularity which actually exhibits the missing mode explicitly. This is obviously not the case for traditional models obtained by projecting the nonlinear equations onto the linear modes, and results have shown that these models may yield quite erroneous simulation results when a seemingly unimportant, higher frequency resonant mode is excluded from them. A technical note was published on this subject in the *Journal of Sound and Vibration* [12].
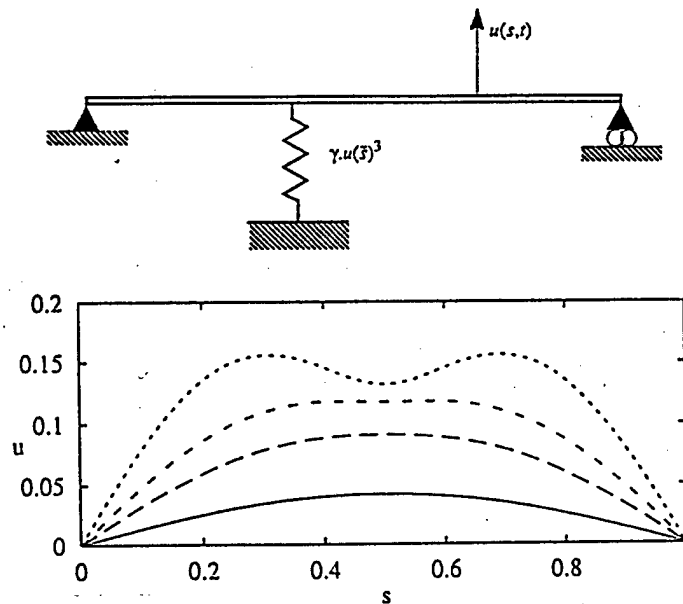
**Figure 1.** Linear beam with a purely nonlinear spring attached (*top*). Beam configuration at peak deflection for four energy levels, with spring located at midspan (*bottom*).
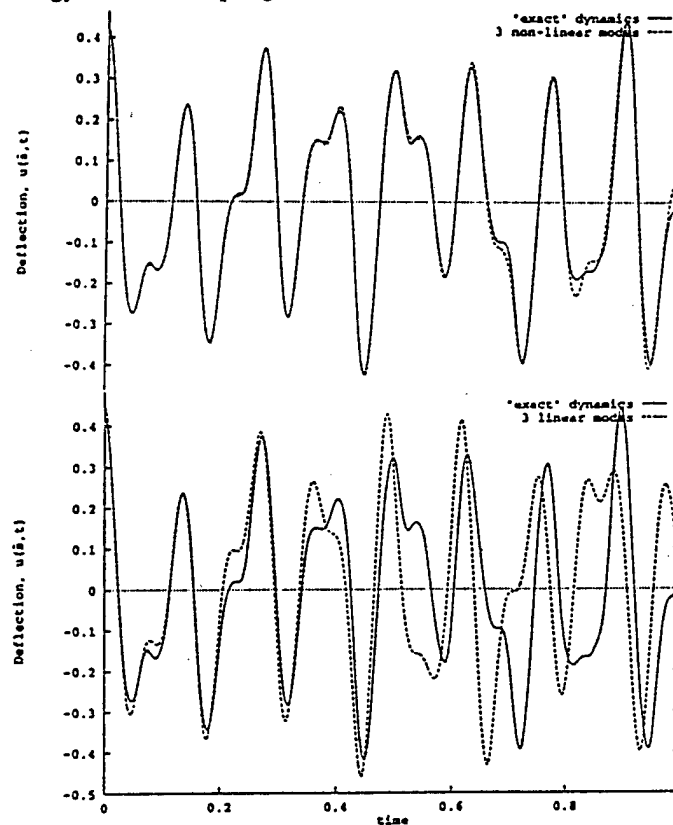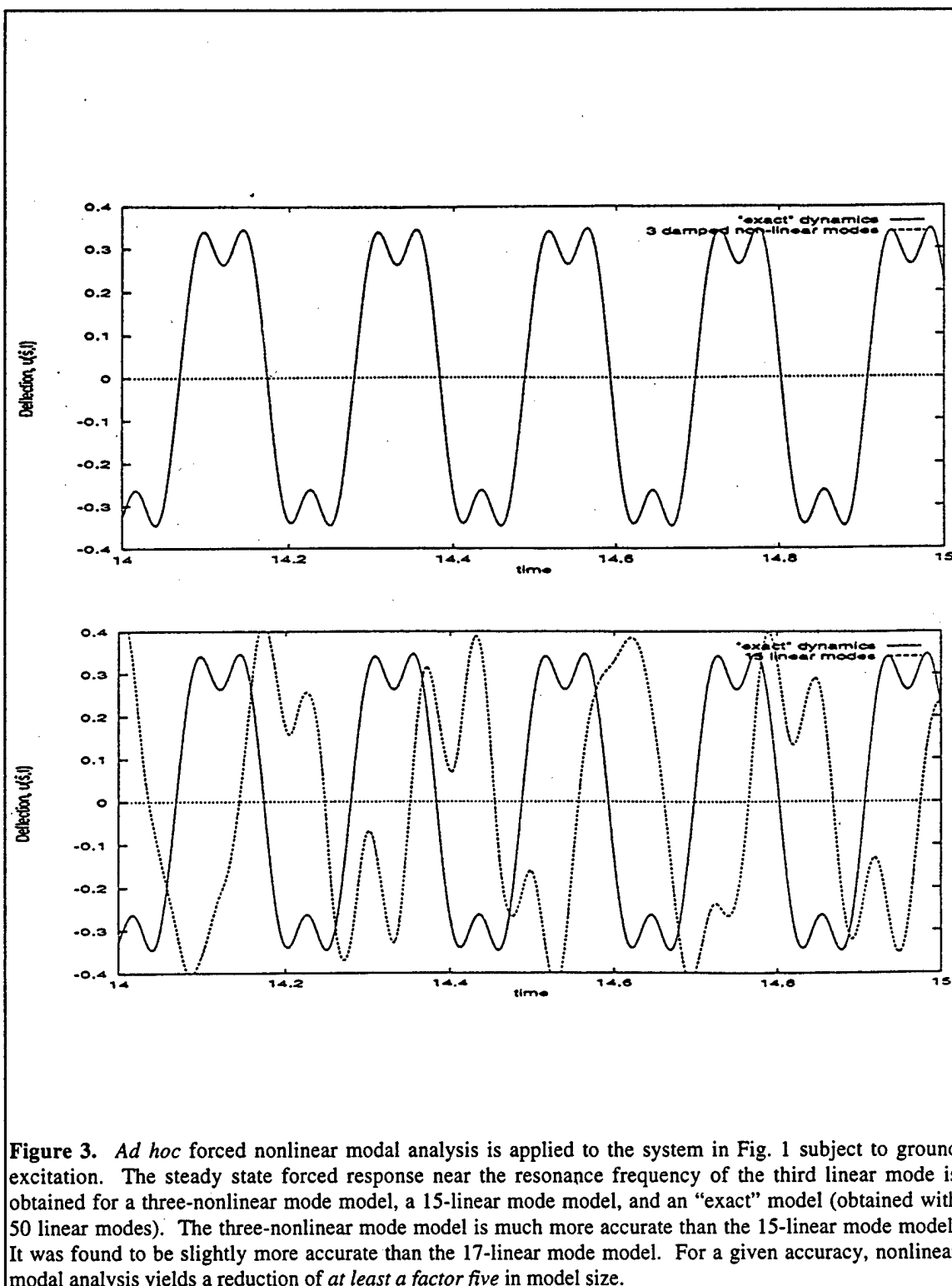


**Figure 2.** Free nonlinear modal analysis is applied to the system in Fig. 1. The response to initial conditions for a three-nonlinear mode model is compared to the "exact" solution (obtained with 50 linear modes), and to the three-linear mode model. It was found that for a given accuracy, nonlinear modal analysis yields a reduction of *at least a factor two* in model size.

**Figure 3.** *Ad hoc* forced nonlinear modal analysis is applied to the system in Fig. 1 subject to ground excitation. The steady state forced response near the resonance frequency of the third linear mode is obtained for a three-nonlinear mode model, a 15-linear mode model, and an "exact" model (obtained with 50 linear modes). The three-nonlinear mode model is much more accurate than the 15-linear mode model. It was found to be slightly more accurate than the 17-linear mode model. For a given accuracy, nonlinear modal analysis yields a reduction of *at least a factor five* in model size.

### Ad Hoc Nonlinear Modal Analysis for Forced Response

A nonlinear modal analysis and associated model reduction procedure were developed for forced motions of nonlinear systems subject to external excitations of small amplitudes. The reduced-order models are based upon the modal manifolds defined for the autonomous system and, thus, become only approximately invariant upon the introduction of small amplitude forcing. Nevertheless, this *ad hoc* formulation was shown to eliminate most of the contamination of and from the modes not included in the reduced-order model, and thus to capture all the important effects of the external excitation and of the internal nonlinear coupling. Results obtained for a study case indicate that this forced nonlinear modal analysis can provide significant improvements, in terms of model reduction, over the traditional linear modal analysis of the nonlinear system, in particular when non-negligible nonlinear modal coupling terms are present. This is illustrated in Fig. 3. The ability to perform a nonlinear modal analysis based on only a few nonlinear modes is very similar in spirit to the techniques existing for the analysis of large-scale linear systems, therefore effectively extending the realm of "modal analysis" to nonlinear systems. The results of the forced response study were written up in references [10] and [13]. (Further journal publications on this topic are forthcoming, based on the results given in [10].)

### Invariant Forced Response Models

The free response methodology was extended to include arbitrary external excitation in an invariant manner. This was achieved by modeling the excitation as the output of an auxiliary linear system, with which the structural dynamic model is augmented, resulting in invariant manifolds that are time-dependent. This extension allows for the consideration of quite generalized excitation terms, including transient forcing, while maintaining invariance properties. Also, this allows one to formulate the problem in an exactly invariant manner for the important case of harmonic excitation, by adding a simple, two-state model for the forcing dynamics.

### Modal Manifolds for Damped systems

While the invariant manifold procedures are very general in nature and can be applied systematically to gyroscopic, non-conservative systems, the determination of the multi-mode manifolds is typically easier for undamped systems than for damped ones. Consequently, it proved beneficial to treat the modal manifolds of a weakly damped system as perturbations of those of its undamped counterpart. For the particular case of proportional linear damping, an even simpler approximation consists of applying the proportional damping directly to the non-linear modal oscillators of the undamped system. These approximations were developed, and were found to facilitate greatly the generation of the reduced-order models for modal analysis [10, 13]. Again, these approximations are very much in the spirit of current techniques for lightly damped large-scale linear systems. (Further journal publications on this topic are forthcoming, based on the results given in [10].)

### Invariant Manifolds for Linear Systems

The concept of invariant manifolds was applied successfully to linear, structural, oscillatory systems. For both discrete and continuous systems, proofs of the equivalence of the normal modes thus obtained were provided by exhibiting explicit transformation relations with the traditional normal modes. Orthogonality relations were derived between the normal modes obtained by the invariant manifold approach, which in turn allow one to perform a modal analysis equivalent to the traditional modal analysis. The invariant manifold formulation of the modes provides a physically insightful, alternate perspective to the traditional definition of normal modes as solutions of an eigenvalue problem. (A full publication on this topic is forthcoming, based on the results given in [10].)

### Computational Implementation

Preliminary work was initiated on the generation of a general computer code for making the method useful for a wide class of problems. This program will allow for the construction of the normal modes, via the appropriate coefficients, from a set of nonlinear system equations with forcing. It will be limited to systems with quadratic and cubic stiffness nonlinearities.

The research carried out during these grant years paves the way for future studies of nonlinear modal analysis. Most notably, research to date has laid a convincing theoretical foundation for nonlinear modal analysis and demonstrated the feasibility of the approach for large-scale structural systems, in particular through the development of a preliminary code for systematic invariant manifold generation. It has thus provided a crucial link in the path to the ultimate aim: incorporation of the methodology into commercial structural dynamics codes and simulation environments.

## PUBLICATIONS FROM ARO SUPPORT

[1] S.W. Shaw and C. Pierre, 1991, "Nonlinear Normal Modes and Invariant Manifolds," *Journal of Sound and Vibration* **150**, 170-173.

[2] S.W. Shaw and C. Pierre, 1992, "On Nonlinear Normal Modes," Invited Paper, in *Nonlinear Vibrations*, DE-Vol. 50, AMD-Vol. 144, edited by R. A. Ibrahim *et al.*, Proceedings of the 1992 ASME Winter Annual Meeting, Anaheim, California.

[3] S.W. Shaw and C. Pierre, 1993, "Normal Modes for Nonlinear Vibratory Systems," *Journal of Sound and Vibration* **164**(1), 85-124.

[4] S.W. Shaw and C. Pierre, 1994, "Normal Modes of Vibration for Nonlinear Continuous Systems," *Journal of Sound and Vibration* **169**(3), 319-347.

[5] C. Pierre and S.W. Shaw, 1991, "Mode Localization Due to Symmetry-Breaking Nonlinearities," *International Journal of Bifurcation and Chaos* **1**, 471-475.

[6] S.R. Hsieh, S.W. Shaw, and C. Pierre, 1994, "Normal Modes for Large Amplitude Motions of a Cantilever Beam," *International Journal of Solids and Structures* **31**(14), 1981-2014.

[7] N. Boivin, C. Pierre, and S.W. Shaw, 1995, "Non-Linear Normal Modes, Invariance, and Modal Dynamics Approximations of Non-Linear Systems," *Nonlinear Dynamics* **8**(3), 1-32.

[8] S.W. Shaw, 1994, "An Invariant Manifold Approach to Nonlinear Normal Modes of Oscillation," *Journal of Nonlinear Science* **4**, 419-448.

[9] S.W. Shaw and C. Pierre, 1995, "Normal Modes and Modal Analysis Techniques for Nonlinear Structural Systems," *Nonlinear Dynamics and Stochastic Mechanics*, CRC Mathematical Modeling Series, 95-119, N. Sri Namachchivaya and W. Kleimann, eds., CRC Press.

[10] N. Boivin, 1995, *Nonlinear Modal Analysis of Structural Systems Using Invariant Manifolds*, Ph.D. Dissertation, Department of Mechanical Engineering and Applied Mechanics, The University of Michigan.

[11] N. Boivin, C. Pierre, and S.W. Shaw, 1994, "Non-Linear Modal Analysis of Structural Systems Using Multi-Mode Invariant Manifolds," AIAA Paper 94-1672, *Proceedings of the AIAA Dynamics Specialists Conference*, Hilton Head, SC.

[12] N. Boivin, C. Pierre, and S.W. Shaw, 1995, "Non-linear Modal Analysis of Structural Systems Featuring Internal Resonances," *Journal of Sound and Vibration* **182**(2), 336-341.

[13] N. Boivin, C. Pierre, and S.W. Shaw, 1996, "Non-linear Modal Analysis of the Forced Response of Non-linear Structural Systems," AIAA Paper 96-1250, *Proceedings of the AIAA Dynamics Specialists Conference*, Salt Lake City, Utah.

[14] S.-L. Chen and S.W. Shaw, 1996, "Normal Modes for Piecewise Linear Vibratory Systems," *Nonlinear Dynamics*, **10**, 135-164.

## SCIENTIFIC PERSONNEL

Christophe Pierre, principal investigator, Professor, Department of Mechanical Engineering and Applied Mechanics, the University of Michigan.

Steven W. Shaw, co-investigator, Professor, Department of Mechanical Engineering, Michigan State University.

Nicolas Boivin, Graduate Student Research Assistant and Doctoral candidate, Department of Mechanical Engineering and Applied Mechanics, the University of Michigan; Awarded Ph.D. in August 1995.

Eric Pesheck, Graduate Student Research Assistant and Doctoral candidate, Department of Mechanical Engineering and Applied Mechanics, the University of Michigan.

## INVENTIONS

None.